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ECE 342: Probability and Statistics **Spring 2025**

Lecture 8.1: Normal/Gaussian Random Variables

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Read BT Chapter 3.3.

Learning Objectives:

- Understand the properties of normal/Gaussian random variables;
- Know how to convert an arbitrary normal random variable to a standard normal random variable.

8.1 Normal Random Variables

Normal / Gaussian random variable X:

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

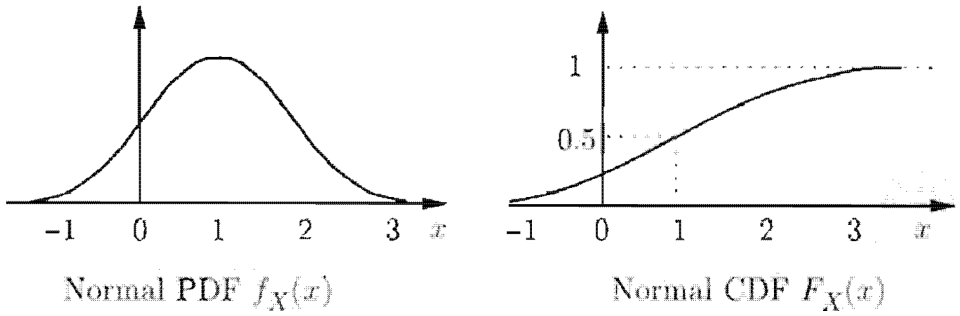

Figure 8.1: PDF and CDF of a normal random variables with $\mu = 1$ and $\sigma^2 = 1$ (Figure 3.9 in the book).

properties of normal random variables:

- $\mathbf{E}[X] = \mu$ and $\text{var}(X) = \sigma^2$
  - a normal random variable's mean and variance completely specifies its statistical properties
- if X is a normal random variable, $Y = aX + b$ is also a normal random variable

standard normal random variable: a normal random variable with zero mean and unit variance

- write the CDF as Φ:

$$\Phi(y) = \mathbf{P}(Y \leq y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{y} e^{-\frac{t^2}{2}} dt$$

- $\Phi(-y) = 1 - \Phi(y)$
- standardize a general normal random variable X: $Y = (X - \mu)/\sigma$
- we can look up values of $\Phi(y)$ from the standard normal table (shown at the end)

Exercises: Examples 3.7 and 3.8, Problems 11–13 in Chapter 3 of BT.

Solution to Problem 11:
(a) X is a standard normal random variable. Therefore, we have

$$\mathbf{P}(X \leq 1.5) = \Phi(1.5) = 0.9332 \text{ and } \mathbf{P}(X \leq -1) = \Phi(-1) = 1 - \Phi(1) = 1 - 0.8413 = 0.1587.$$

(b) $(Y - 1)/2$ is a standard normal random variable.
(c) We have

$$\begin{aligned} \mathbf{P}(-1 \leq Y \leq 1) &= \mathbf{P}(-1 \leq (Y-1)/2 \leq 0) \\ &= \Phi(0) - \Phi(-1) = \Phi(0) - [1 - \Phi(1)] = 0.5 - (1 - 0.8413) = 0.3413. \end{aligned}$$

Solution to Problem 12:
The random variable X/σ is a standard normal random variable. Therefore, we have

$$\mathbf{P}(X \geq k\sigma) = \mathbf{P}(X/\sigma \geq k) = 1 - \Phi(k).$$

From the standard normal table, we get

$$\mathbf{P}(X \geq \sigma) = 0.1587, \quad \mathbf{P}(X \geq 2\sigma) = 0.0228, \quad \mathbf{P}(X \geq 3\sigma) = 0.0014.$$

Similarly, we have

$$\mathbf{P}(|X| \leq k\sigma) = \mathbf{P}(|X/\sigma| \leq k) = \mathbf{P}(-k \leq X/\sigma \leq k) = \Phi(k) - \Phi(-k) = \Phi(k) - [1 - \Phi(k)] = 2\Phi(k) - 1.$$

From the standard normal table, we get

$$\mathbf{P}(|X| \leq \sigma) = 0.6826, \quad \mathbf{P}(|X| \leq 2\sigma) = 0.9544, \quad \mathbf{P}(|X| \leq 3\sigma) = 0.9972.$$

Solution to Problem 13:
First, note that 59 degrees Fahrenheit is $(59 - 32) \times \frac{5}{9} = 15$ degrees celsius. Let X be the temperature in Celsius. We need to calculate $\mathbf{P}(X \leq 15)$. We have

$$\mathbf{P}(X \leq 15) = \mathbf{P}\left(\frac{X - 10}{10} \leq \frac{15 - 10}{10}\right) = \mathbf{P}\left(\frac{X - 10}{10} \leq 0.5\right) = \Phi(0.5) = 0.6915.$$

	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Figure 8.2: The standard normal table.