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ECE 342: Probability and Statistics **Spring 2025**

Lecture 6.1: Multiple Random Variables

Lecturer: Yuanzhang Xiao

Read BT Chapter 2.5.

6.1 Joint PMFs of Multiple Random Variables

in the previous chapter, we learned about probabilities of two and multiple events

similarly, we are interested in joint PMFs of multiple random variables

given two discrete random variables X and Y, their joint PMF is

$$p_{X,Y}(x,y) = \mathbf{P}\left(X = x, Y = y\right),$$

where $\mathbf{P}\left(X = x, Y = y\right)$ is the shorthand for $\mathbf{P}\left(\{X = x\} \cap \{Y = y\}\right)$

using joint PMF, we can calculate

- probability of event A that can be specified by properties of X and Y

$$\mathbf{P}\left((X,Y) \in A\right) = \sum_{(x,y)\in A} p_{X,Y}(x,y)$$

- marginal PMFs of X or Y

$$p_X(x) = \sum_y p_{X,Y}(x,y) \quad \text{and} \quad p_Y(y) = \sum_x p_{X,Y}(x,y)$$

6.1.1 Functions of Multiple Random Variables

a function of random variables X and Y, $Z = g(X,Y)$, is also a random variable

PMF of Z is

$$p_Z(z) = \sum_{\{(x,y)|g(x,y)=z\}} p_{X,Y}(x,y)$$

the expectation of Z is

$$\mathbf{E}[Z] = \mathbf{E}\left[g(X,Y)\right] = \sum_x \sum_y g(x,y) p_{X,Y}(x,y)$$

a special case when g is linear,

$$\mathbf{E}\left[aX + bY + c\right] = a\mathbf{E}\left[X\right] + b\mathbf{E}\left[Y\right] + c$$

Exercises: Example 2.9 in Chapter 2 of BT.

6.1.2 More than Two Random Variables

all the above discussions generalize to more than two random variables

- for example, joint PMF of three random variables

$$p_{X,Y,Z}(x,y,z) = \mathbf{P}(X = x, Y = y, Z = z)$$

- marginal PMFs

$$p_{X,Y}(x,y) = \sum_z p_{X,Y,Z}(x,y,z)$$

  and

$$p_X(x) = \sum_y \sum_z p_{X,Y,Z}(x,y,z)$$

- expectation of functions

$$\mathbf{E}\left[g(X,Y,Z)\right] = \sum_x \sum_y \sum_z g(x,y,z) p_{X,Y,Z}(x,y,z)$$

- expectationn of linear functions

$$\mathbf{E}\left[a_1 X_1 + a_2 X_2 + \cdots + a_n X_n\right] = a_1 \mathbf{E}\left[X_1\right] + a_2 \mathbf{E}\left[X_2\right] + \cdots + a_n \mathbf{E}\left[X_n\right]$$

Exercises: Examples 2.10 and 2.11 in Chapter 2 of BT.