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ECE 342: Probability and Statistics **Spring 2025**

Lecture 5.2: Expectation and Variance

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Read BT Chapter 2.4.

5.1 Expectation and Variance

although PMF is comprehensive, summary statistics are often sufficient and convenient

5.1.1 Expectation

expectation / mean of a random variable X is

$$\mathbb{E}[X] = \sum_x x p_X(x)$$

Exercises: Example 2.2 in Chapter 2 of BT.

5.1.2 Variance and Moments

variance of a random variable X is

$$\text{var}(X) = \mathbb{E}\left[(X - \mathbb{E}[X])^2\right]$$

standard deviation of a random variable X is

$$\sigma_X = \sqrt{\text{var}(X)}$$

which is easier to interpret since it is in the same unit as X

n-th moment of a random variable X is

$$\mathbb{E}[X^n]$$

5.1.3 Expected Value Rule

expected value rule:

$$\mathbb{E}[g(X)] = \sum_x g(x) p_X(x)$$

expected value rule provides an easy way to calculate the variance:

$$\text{var}(X) = \sum_x (x - \mathbb{E}[X])^2 p_X(x)$$

Exercises: Example 2.3 in Chapter 2 of BT.

5.1.4 Properties of Mean and Variance

for a linear function of a random variable, namely $Y = aX + b$, we have

$$\mathbb{E}[Y] = a\mathbb{E}[X] + b, \text{ and } \text{var}(Y) = a^2\text{var}(X)$$

this property does not hold for a general function, i.e., $\mathbb{E}[g(X)] \neq g(\mathbb{E}[X])$ in general

Exercises: Example 2.4 in Chapter 2 of BT.

we can calculate the variance using moments:

$$\text{var}(X) = \mathbb{E}\left[X^2\right] - (\mathbb{E}[X])^2$$

so far, we have 3 ways of calculating the variance:

- using the definition: treat $(X - \mathbb{E}[X])^2$ as a random variable and take its expectation
- using expected value rule:
$$\text{var}(X) = \sum_x (x - \mathbb{E}[X])^2 p_X(x)$$
- using moments:
$$\text{var}(X) = \mathbb{E}\left[X^2\right] - (\mathbb{E}[X])^2$$

Exercises: Examples 2.5–2.8 in Chapter 2 of BT.

5.2 Exercises on PMF, Expectation, and Variance

Exercises: Problems 19, 21, 22 and 23 in Chapter 2 of BT.

Data scientist interview questions:

JP Morgan: You are playing a game where you roll a 6-sided die up to two times and can choose to stop following the first roll if you wish. You will receive a dollar amount equal to the final amount rolled. How much are you willing to pay to play this game?

Morgan Stanley: Players A and B play the following game: a number k from 1–6 is chosen, and A and B will toss a die until the first person throws a die showing side k, after which that person is awarded \$100 and the game is over.

- How much is player A willing to pay to play first in this game?
- How much is player A willing to pay to play second in this game?