.1

ECE 342: Probability and Statistics **Spring 2025**

Lecture 4.1: Independence

Lecturer: Yuanzhang Xiao

Fun Fact: *If Gauss finds no correlation, they are independent.*

Read BT Chapter 1.5.

4.1 Independence

4.1.1 Independence of Two Events

Two events / sets A and B are **independent** if the occurrence of one event provides no information about the occurrence of the other.

Two equivalent mathematical definitions of independence:

- A is independent of B if $\mathbf{P}(A|B) = \mathbf{P}(A)$, assuming that $\mathbf{P}(B) > 0$
- A is independent of B if $\mathbf{P}(A \cap B) = \mathbf{P}(A) \cdot \mathbf{P}(B)$

some properties of independence:

- symmetry: A is independent of $B \Leftrightarrow B$ is independent of $A \Leftrightarrow A$ and B are independent
- if A and B are independent, then A and B^c are independent
- two disjoint sets A and B are generally *not* independent, unless at least one of them is empty

Exercises: Examples 1.19 in Chapter 1 of BT.

4.1.2 Conditional Independence

Given an event C, events A and B are **conditionally independent** if

$$\mathbf{P}(A \cap B|C) = \mathbf{P}(A|C) \cdot \mathbf{P}(B|C)$$

Equivalent definition:

$$\mathbf{P}(A|B \cap C) = \mathbf{P}(A|C)$$

physical meaning: if C occurs, the knowledge that B occurs does not change the probability of A

independence does not imply conditional independence, and vice versa

Exercises: Examples 1.20, 1.21 in Chapter 1 of BT.

4.1.3 Independence of a Collection of Events

events $A_1, A_2, \ldots, A_n$ are independent if

$$\mathbf{P}\left(\cap_{i \in S} A_i\right) = \prod_{i \in S} \mathbf{P}(A_i)$$

for every subset S **of** $\{1, 2, \ldots, n\}$

For example, for three events A_1, A_2, A_3 to be inndependent, we need

$$\begin{aligned}
\mathbf{P}(A_1 \cap A_2) &= \mathbf{P}(A_1) \cdot \mathbf{P}(A_2) \\
\mathbf{P}(A_1 \cap A_3) &= \mathbf{P}(A_1) \cdot \mathbf{P}(A_3) \\
\mathbf{P}(A_2 \cap A_3) &= \mathbf{P}(A_2) \cdot \mathbf{P}(A_3) \\
\mathbf{P}(A_1 \cap A_2 \cap A_3) &= \mathbf{P}(A_1) \cdot \mathbf{P}(A_2) \cdot \mathbf{P}(A_3)
\end{aligned}$$

pairwise independence is not enough for independence

Exercises: Examples 1.22–1.23 in Chapter 1 of BT.