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ECE 342: Probability and Statistics **Spring 2025**

Module 3.3: Bayes' Rule

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Fun Fact: *Gauss is neither a frequentist nor a Bayesian. For gauss, the probability is always 1.*

Read BT Chapter 1.4.

3.1 Bayes' Rule and Inference

Bayes' rule: Let $A_1, \ldots, A_n$ be the disjoint events that form a partition of Ω and assume that $\mathbf{P}(A_i) > 0$ for all i. Then for any event B such that $\mathbf{P}(B) > 0$, we have

$$\mathbf{P}(A_i|B) = \frac{\mathbf{P}(A_i) \cdot \mathbf{P}(B|A_i)}{\mathbf{P}(A_1) \cdot \mathbf{P}(B|A_1) + \cdots + \mathbf{P}(A_n) \cdot \mathbf{P}(B|A_n)}.$$

Bayes' rule can be used for inference:

- A_i: causes
- B: effect / observation
- $\mathbf{P}(A_i)$: prior probability
- $\mathbf{P}(A_i|B)$: posterior probability (inference on the causes after observing the effect)

Exercises: Examples 1.16–1.18, Problem 19 in Chapter 1 of BT.

An additional exercise problem that was a popular Facebook interview question:
Suppose that the probability of raining in Seattle is 0.1 in any given day. You are going to Seattle for an interview and need to decide whether to bring an umbrella. You have 3 friends in Seattle and ask them. Each one will tell you the truth with probability 2/3 and lie with probability 1/3. If they all say that it is raining, what is the probability that it is actually raining?