

## Module 3.2: Total Probability Theorem

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Read BT Chapter 1.4.

## 3.1 Total Probability Theorem

**Total Probability Theorem:** Let  $A_1, \dots, A_n$  be the disjoint events that form a partition of  $\Omega$  and assume that  $\mathbf{P}(A_i) > 0$  for all  $i$ . Then for any event  $B$ , we have

$$\mathbf{P}(B) = \mathbf{P}(A_1) \cdot \mathbf{P}(B|A_1) + \dots + \mathbf{P}(A_n) \cdot \mathbf{P}(B|A_n).$$

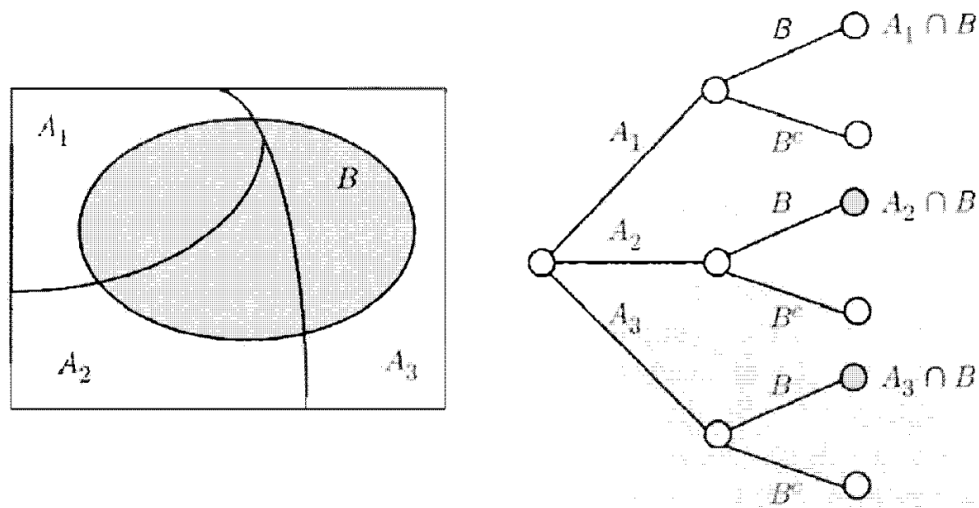


Figure 3.1: Illustration of total probability theorem (Figure 1.13 in the book).

The total probability theorem provides a “divide-and-conquer” approach of calculating  $\mathbf{P}(B)$ .

The key is to choose a partition  $A_1, \dots, A_n$  such that  $\mathbf{P}(B|A_i)$  are easy to compute.

**Exercises:** Examples 1.13–1.15 in Chapter 1 of BT.