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ECE 342: Probability and Statistics **Spring 2025**

Module 3.1: Conditional Probability and Multiplication Rule

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Fun Fact: *Gauss can divide by zero.*

Read BT Chapter 1.3.

3.1 Conditional Probability

3.1.1 Definition

Conditional probability – reasoning about uncertainty based on **partial information**.

- two die rolls: given that the sum is 9, how likely is that the 1st roll is 6?
- drug testing: how likely a person uses a drug given that the test result is negative?

conditional probability of A given B: $\mathbf{P}(A|B)$

how to calculate conditional probabilities?

- In the example of two die rolls, the outcomes with the sum being 9 are $\{(3,6),(4,5),(5,4),(6,3)\}$. Since all outcomes are equally likely, we have

$$\mathbf{P}\left(\text{the 1st roll is 6} \mid \text{the sum is 9}\right) = \frac{1}{4}$$

- When all outcomes are equally likely, we have

$$\mathbf{P}(A|B) = \frac{\text{number of elements in } A \cap B}{\text{number of elements in } B}$$

- In general, we have

$$\mathbf{P}(A|B) = \frac{\mathbf{P}(A \cap B)}{\mathbf{P}(B)}, \text{ assuming } \mathbf{P}(B) > 0$$

the conditional probability defined above obeys probability axioms (nonnegativity, additivity, normalization)

therefore, conditional probabilities also satisfy properties of probability laws key steps in calculating $\mathbf{P}(A|B)$:

1. identify the sample space Ω
2. identify A, B, and $A \cap B$
3. calculate $\mathbf{P}(A|B)$:
   - if the outcomes are equally likely, use
   $$\mathbf{P}(A|B) = \frac{\text{number of elements in } A \cap B}{\text{number of elements in } B}$$
   - if the outcomes are not equally likely, use
   $$\mathbf{P}(A|B) = \frac{\mathbf{P}(A \cap B)}{\mathbf{P}(B)}$$

Exercises: Examples 1.6–1.8 in Chapter 1 of BT.

3.1.2 Using Conditional Probability in Sequential Experiments

The definition of conditional probability implies that

$$\mathbf{P}(A \cap B) = \mathbf{P}(B) \cdot \mathbf{P}(A|B)$$

The above rule can be generalized as

$$\mathbf{P}\left(\cap_{i=1}^{n} A_i\right) = \mathbf{P}(A_1) \cdot \mathbf{P}(A_2|A_1) \cdot \mathbf{P}(A_3|A_1 \cap A_2) \cdots \mathbf{P}\left(A_n | \cap_{i=1}^{n-1} A_i\right)$$

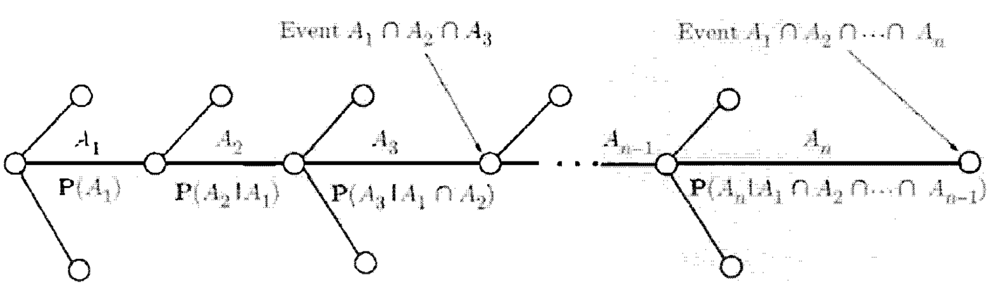

Figure 3.1: Illustration of the multiplication rule (Figure 1.10 in the book).

calculating probabilities in a sequential experiment:

1. draw a tree-based sequential description of the experiment
2. identify the leaf associated the event of interest
3. identify the branch that starts from the root and ends at the leaf
4. identify the conditional probabilities along the branch
5. use the multiplication rule to compute the probability

Exercises: Examples 1.9–1.12 in Chapter 1 of BT.