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EE 342: Probability and Statistics **Spring 2025**

Module 2.2: Probability Laws

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Fun Fact: *Gauss can prove axioms.*

Read BT Chapter 1.2.

2.0.1 How to Define a Probability Law?

Probability Axioms:

1. **Nonnegativity**: $\mathbf{P}(A) \geq 0$ for every event A
2. **Additivity**: if A and B are disjoint, we have
$$\mathbf{P}(A \cup B) = \mathbf{P}(A) + \mathbf{P}(B);$$
if $A_1, A_2, \ldots$ are disjoint, we have
$$\mathbf{P}(A_1 \cup A_2 \cup \cdots) = \mathbf{P}(A_1) + \mathbf{P}(A_2) + \cdots .$$
3. **Normalization**: $\mathbf{P}(\Omega) = 1$

2.1 Probability Laws

2.1.1 Properties of Probability Laws

Most properties of probability laws can be derived from the three probability axioms.

- $\mathbf{P}(\phi) = 0$
- if $A \subset B$, then $\mathbf{P}(A) \leq \mathbf{P}(B)$
- $\mathbf{P}(A \cup B) = \mathbf{P}(A) + \mathbf{P}(B) - \mathbf{P}(A \cap B)$
- $\mathbf{P}(A \cup B) \leq \mathbf{P}(A) + \mathbf{P}(B)$; in general, $\mathbf{P}(A_1 \cup A_2 \cup \cdots \cup A_n) \leq \sum_{i=1}^{n} \mathbf{P}(A_i)$
- $\mathbf{P}(A \cup B \cup C) = \mathbf{P}(A) + \mathbf{P}(A^c \cap B) + \mathbf{P}(A^c \cap B^c \cap C)$

Exercises: Problems 9–10 in Chapter 1 of BT.

We can illustrate all the above properties by Venn diagrams.

Now the question is how to calculate probabilities?

2.1.2 Calculating Probabilities in Discrete Models

In discrete models, the universal set Ω has a finite number of elements/outcomes.

Discrete Probability Law

$$\mathbf{P}(\{s_1, s_2, \ldots, s_n\}) = \mathbf{P}(s_1) + \mathbf{P}(s_2) + \cdots + \mathbf{P}(s_n),$$

where we write $\mathbf{P}(s_i)$ for $\mathbf{P}(\{s_i\})$ hereafter.

In the special case where all n outcomes are equally likely, we have

$$\mathbf{P}(A) = \frac{\text{number of elements of } A}{n}$$

Exercises: Examples 1.2–1.3 and Problems 5–8 in Chapter 1 of BT.

2.1.3 Calculating Probabilities in Continuous Models

In continuous models, the universal set has infinitely many elements/outcomes.

The probability of an event is the "length" or "area" of the set.

Exercises: Examples 1.4–1.5 in Chapter 1 of BT.