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EE 342: Probability and Statistics **Spring 2025**

Module 2.1: Probabilistic Models

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Fun Fact: *Gauss can recite all of π, backwards.*

Read BT Chapter 1.2.

2.1 Probabilistic Models

A **probabilistic model** is mathematical description of an uncertain situation.

Elements of a probabilistic model:

1. **sample space** Ω: the set of all possible outcomes of an experiment
2. **probability law**: assigns a set $A \subset \Omega$ of possible outcomes (i.e., an **event**) a nonnegative number $\mathbf{P}(A)$ (i.e., **probability** of A)

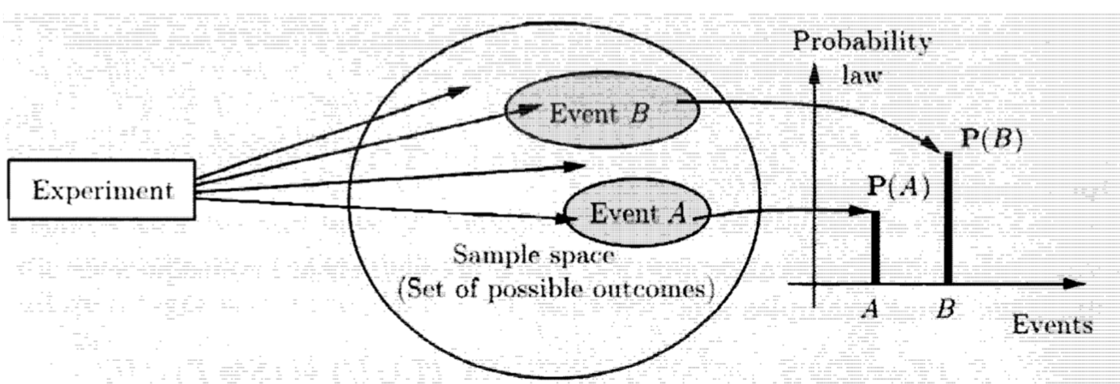

Figure 2.1: Elements of a probabilistic model (Figure 1.2 in the book).

An **experiment** has **outcomes**, all of which are collected in the **sample space**, whose subsets are **events**.

- outcomes $\leftrightarrow$ elements of the set
- events $\leftrightarrow$ sets

An illustrative example:

- experiment: a die roll
- 6 outcomes: 1, 2, 3, 4, 5, 6
- sample space $\{1, 2, 3, 4, 5, 6\}$
- event A: "the roll is an even number", $A = \{2, 4, 6\}$; event B: "the roll is smaller than 3", $B = \{1, 2\}$

Another example:

- experiment: a die roll
- 3 outcomes: "2 or 4", "1 or 3 or 5", "6"
- sample space {"2 or 4", "1 or 3 or 5", "6"}
- event A: "the roll is an even number", $A =$ {"2 or 4", "6"}; event B: "the roll is smaller than 3", does not exist in this model

Take-away messages:

- the definition of a probabilistic model is *flexible*
- the way we defines a probabilistic model (e.g., outcomes) matters a lot
- some definition works "better" than others (e.g., existence of certain events of interest)

2.1.1 How to Define a Sample Space?

Requirements for a properly defined sample space:

- elements of the sample space must be **distinct/mutually exclusive**
  - example of distinct elements: "2 or 4", "1 or 3 or 5", "6"
  - example of non-distinct elements: "2 or 4", "1 or 3 or 5", "4 or 6"
  - we would not have probability axioms (described next) if we allowed non-distinct elements
- the sample space must be **exhaustive** (i.e., contains all outcomes)

Proper definition of a sample space is sometimes an art – simple yet sufficiently detailed for our purpose.

BT Example 1.1: ten consecutive coin tosses

- Game 1: \$1 for each head
- Game 2: \$1 for each coin toss up to the first head, \$2 for each coin toss up to the second head, \$4 for each coin toss up to the third head; dollar amount doubled after each head

Game 1:

- 11 outcomes: number of heads
- sample space: $\{0, 1, 2, \ldots, 10\}$

Game 2:

- $2^{10} = 1024$ outcomes: sequence of toss results
- sample space: $\{HHH \cdots H, HHH \cdots T, \ldots, TTT \cdots T\}$

When experiments are sequential, it may be convenient to use a tree-based sequential description.

- it is natural to use conditional probabilities and total probability law on tree-based description

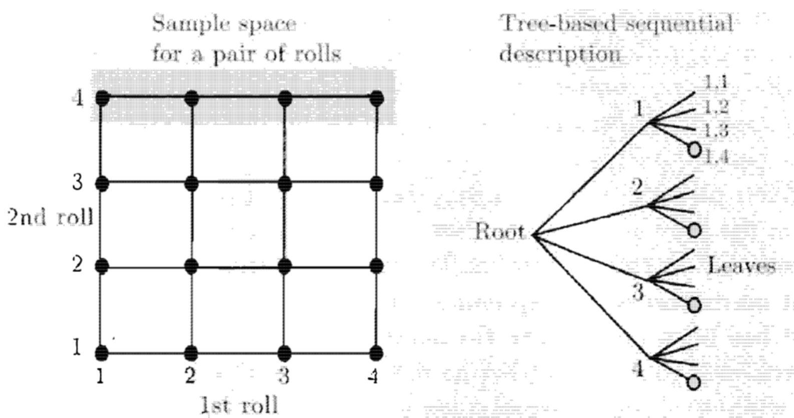

Figure 2.2: Two equivalent descriptions of the sample space (Figure 1.3 in the book).