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ECE 342: Probability and Statistics **Spring 2025**

Module 1.2: Sets and Algebra of Sets

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Fun Fact: *An empty set is the set of theorems that Gauss cannot prove.*

Read BT Chapter 1.1.

1.1 Sets

1.1.1 Definitions and Basics

Definition 1.1 *A **set** is a collection of objects. We also refer to the objects as **elements** of the set.*

Some useful notations and examples:

- $x \in S$: element x is in set S; $x \notin S$: element x is not in set S
- empty set ϕ: a set with no elements
- a set with a finite number of elements:
$$S = \{x_1, x_2, \ldots, x_n\}.$$
  - set of outcomes of a dice roll: $S = \{1, 2, 3, 4, 5, 6\}$
  - set of outcomes of a coin toss: $S = \{H, T\}$
- a set with infinitely many elements $S = \{x_1, x_2, \ldots\}$
- a set defined by properties P:
$$S = \{x \mid x \text{ satisfies } P\}$$
  - set of even integers: $S = \{k \mid k/2 \text{ is integer}\}$
  - set of real numbers in $[0, 1]$: $S = \{x \mid 0 \leq x \leq 1\}$
- S is a **subset** of T: $S \subset T$ (i.e., every element in S is an element of T)
- $S = T$ if and only if $S \subset T$ and $T \subset S$.
- universal set Ω: a set containing all the objects of interest

1.1.2 Set Operations

A list of useful operations on sets:

- complemenet of S: $S^c = \{x \in \Omega \mid x \notin S\}$
  - $\Omega^c = \phi$
- union: $S \cup T = \{x \mid x \in S \text{ or } x \in T\}$
  - in general, $\cup_{n=1}^{\infty} S_n = S_1 \cup S_2 \cup \cdots = \{x \mid x \in S_n \text{ for some } n\}$
- intersection: $S \cap T = \{x \mid x \in S \text{ and } x \in T\}$
  - in general, $\cap_{n=1}^{\infty} S_n = S_1 \cap S_2 \cap \cdots = \{x \mid x \in S_n \text{ for all } n\}$
- two sets are disjoint if $S \cap T = \phi$
- partition of S: a collection of disjoint sets whose their union is S
- ordered pair of elements/objects: (x, y)
- set of scalars $\mathbb{R}$; two-dimensional space $\mathbb{R}^2$

Venn diagram is a good way to visualize set operations.

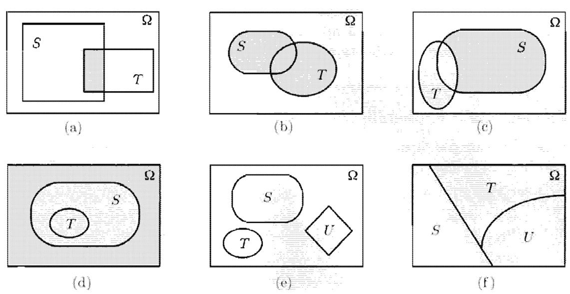

Figure 1.1: Venn diagram (Figure 1.1. in the book).

1.1.3 Algebra of Sets

Some properties of set operations:

- $S \cup T = T \cup S$; $S \cap T = T \cap S$
  - "commutative law"
- $(S \cup T) \cup U = S \cup (T \cup U)$; $(S \cap T) \cap U = S \cap (T \cap U)$
  - "associative law"
  - note: $S \cap (T \cup U) \neq (S \cap T) \cup U$; the reason is below $\downarrow$
- $S \cap (T \cup U) = (S \cap T) \cup (S \cap U)$; $S \cup (T \cap U) = (S \cup T) \cap (S \cup U)$

  - "distributive law"
- $(S^c)^c = S$
- $S \cap S^c = \phi$
- $S \cup \Omega = \Omega$
- $S \cap \Omega = S$
- De Morgan's law
$$(\cup_n S_n)^c = \cap_n S_n^c$$
$$(\cap_n S_n)^c = \cup_n S_n^c$$
  - useful when it is easier to calculate the probability of event S_n^c as opposed to event S_n

We can "prove" the above by Venn diagram.

Exercises: Problems 1–3 in Chapter 1 of BT.