$$\alpha_1 \underline{v} + \alpha_2 \underline{w} + \alpha_3 \underline{z} = \begin{bmatrix} | & | & | \\ \underline{v} & \underline{w} & \underline{z} \\ | & | & | \end{bmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix}$$

cols in first matrix = # rows in second matrix

$$\beta_1 \underline{v} + \beta_2 \underline{w} + \beta_3 \underline{z} \qquad \begin{bmatrix} | & | & | \\ \underline{v} & \underline{w} & \underline{z} \\ | & | & | \end{bmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix}$$

To keep track of both linear combinations of $\underline{v}$, $\underline{w}$, $\underline{z}$,

$$\begin{pmatrix} | & | & | \\ \underline{v} & \underline{w} & \underline{z} \\ | & | & | \end{pmatrix} \begin{pmatrix} \alpha_1 & \beta_1 \\ \alpha_2 & \beta_2 \\ \alpha_3 & \beta_3 \end{pmatrix} = \begin{pmatrix} | & | \\ \alpha_1 \underline{v} + \alpha_2 \underline{w} & \beta_1 \underline{v} + \beta_2 \underline{w} \\ + \alpha_3 \underline{z} & + \beta_3 \underline{z} \\ | & | \end{pmatrix}$$

$$\underline{v} = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} \qquad \underline{w} = \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix} \qquad \underline{z} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

(i)

$$\underbrace{\begin{pmatrix} -1 & 3 & 1 \\ 2 & 0 & 1 \\ 3 & -1 & 1 \end{pmatrix}}_{L} \underbrace{\begin{pmatrix} 1 & 1 \\ 1 & 1 \\ 0 & -2 \end{pmatrix}}_{R} = \begin{pmatrix} 1 \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} + 1 \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix} + 0 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} & 1 \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} + 1 \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix} + (-2) \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 0 \\ 2 & 0 \\ 2 & 0 \end{pmatrix}.$$

$$1 \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} + 1 \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix} + -2 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix} + \begin{pmatrix} -2 \\ -2 \\ -2 \end{pmatrix}$$

$$= \begin{pmatrix} -1 + 3 - 2 \\ 2 + 0 - 2 \\ 3 - 1 - 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}.$$

$$\underbrace{\begin{pmatrix} -1 & 3 & 1 \\ 2 & 0 & 1 \\ 3 & -1 & 1 \end{pmatrix}}_{L} \underbrace{\begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}}_{R'} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$AB = AC$ does NOT mean $B = C$

$AB - AC = 0$

$A(B - C) = 0.$

$$A \underbrace{[\, b_1 \;\; b_2 \;\; b_3 \; \ldots \; b_p \,]}_{B} = [\, Ab_1 \;\; \underset{AB}{Ab_2} \; \ldots \; Ab_p \,].$$

$$\underbrace{\begin{pmatrix} -1 & 3 & 1 \\ 2 & 0 & 1 \\ 3 & -1 & 1 \end{pmatrix}}_{L} \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ -2 & 0 & 0 & 2 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 3 & -1 & 2 & 2 & 1 \\ 0 & 0 & 2 & 2 & 2 & 1 \\ 0 & -1 & 3 & 2 & 2 & 1 \end{pmatrix}$$

3rd col.

$$\begin{array}{r} \\ 2^{nd} \text{ row} \\ \\ \end{array} \begin{pmatrix} -1 & 3 & 1 \\ 2 & 0 & 1 \\ 3 & -1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 3 & 1 & -1 \\ 0 & 1 & 2 \\ -1 & 1 & 3 \end{pmatrix}$$

2nd row 3rd col

$$\begin{pmatrix} -1 & 3 & 1 \\ 2 & 0 & 1 \\ 3 & -1 & 1 \end{pmatrix} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{identity. } I_{3\times3}} = \begin{pmatrix} -1 & 3 & 1 \\ 2 & 0 & 1 \\ 3 & -1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{pmatrix} + \begin{pmatrix} -1 & 2 \\ 0 & -2 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 3 \\ 2 & -1 \\ 5 & 1 \end{pmatrix}$$

$$\alpha \begin{pmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} \alpha & \alpha \\ 2\alpha & \alpha \\ 3\alpha & \alpha \end{pmatrix} \qquad \begin{pmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{pmatrix} \underbrace{\begin{pmatrix} \alpha & 0 \\ 0 & \alpha \end{pmatrix}}_{\alpha I_{2\times2}} = \begin{pmatrix} \alpha & \alpha \\ 2\alpha & \alpha \\ 3\alpha & \alpha \end{pmatrix}$$

$$\begin{array}{cc} A & B \\ m \times n & n \times p \\ m \text{ rows}, & n \text{ rows} \\ n \text{ cols}. & p \text{ cols}. \end{array}$$

$$\underset{m \times p}{\underset{m \times n}{A}\ \underset{n \times p}{B}}$$

$$\underset{n \times p}{B} \quad \underset{m \times n}{A}.$$

in general $AB \neq BA$

Distributive law: $A(B+C) = AB + AC$

$(A+B)C = AC + BC$

Associative law: $(AB)C = A(BC)$

$\neq A(CB)$

$$A_1((A_2\ (A_3\ A_4))\ A_5)$$

In particular $A\underline{x}$ = linear comb of cols of A.

$$A = \begin{pmatrix} \boxed{-1} & \boxed{3} & \boxed{1} \\ 2 & 0 & 1 \\ 3 & -1 & 1 \end{pmatrix} \qquad \underline{b}_1 = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} \qquad \underline{b}_2 \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}.$$

Is $\underline{b}_1$ a linear combination of $\begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$, $\begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

Is $\underline{b}_2$ a — " ——————— " ———

Solve $A\underline{x} = \underline{b}_2$

If there is a soln, $\underline{b}_2$ is a linear comb of cols of A.

If there is NO soln, $\underline{b}_2$ is NOT a —" ————→