

$$\alpha_1 \underline{v} + \alpha_2 \underline{w} + \alpha_3 \underline{z} = \begin{bmatrix} | & | & | \\ \underline{v} & \underline{w} & \underline{z} \\ | & | & | \end{bmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix}$$

# cols in first matrix = # rows in second matrix

$$\beta_1 \underline{v} + \beta_2 \underline{w} + \beta_3 \underline{z} = \begin{bmatrix} | & | & | \\ \underline{v} & \underline{w} & \underline{z} \\ | & | & | \end{bmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix}$$

To keep track of both linear combinations of  $\underline{v}, \underline{w}, \underline{z}$ ,

$$\begin{pmatrix} | & | & | \\ \underline{v} & \underline{w} & \underline{z} \\ | & | & | \end{pmatrix} \begin{pmatrix} \alpha_1 & \beta_1 \\ \alpha_2 & \beta_2 \\ \alpha_3 & \beta_3 \end{pmatrix} = \begin{pmatrix} \alpha_1 \underline{v} + \alpha_2 \underline{w} & \beta_1 \underline{v} + \beta_2 \underline{w} \\ + \alpha_3 \underline{z} & + \beta_3 \underline{z} \\ | & | \end{pmatrix}$$

$$\underline{v} = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} \quad \underline{w} = \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix} \quad \underline{z} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$(i) \quad \left( \begin{array}{ccc|ccc} -1 & 3 & 1 & 1 & 0 & 1 \\ 2 & 0 & 1 & 1 & 1 & 1 \\ 3 & -1 & 1 & 0 & -2 & 1 \end{array} \right) \left( \begin{array}{cc|c} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & -2 & 1 \end{array} \right) = \left( \begin{array}{c} \underline{v} \\ \underline{w} \\ \underline{z} \end{array} \right)$$

$$= \left( \begin{array}{c} \underline{v} \\ \underline{w} \\ \underline{z} \end{array} \right) = \left( \begin{array}{c} \underline{v} \\ \underline{w} \\ \underline{z} \end{array} \right)$$

$$\textcircled{1} \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} + 1 \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix} + \textcircled{-2} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix} + \begin{pmatrix} -2 \\ -2 \\ -2 \end{pmatrix}$$

$$= \begin{pmatrix} -1 + 3 - 2 \\ 2 + 0 - 2 \\ 3 - 1 - 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\left( \begin{array}{c|c|c} -1 & 3 & 1 \\ 2 & 0 & 1 \\ 3 & -1 & 1 \end{array} \right) \left( \begin{array}{c} 1 \\ 1 \\ -2 \end{array} \right) = \left( \begin{array}{c} 0 \\ 0 \\ 0 \end{array} \right)$$

$L$        $R'$

$AB = AC$  does NOT mean  $B = C$   
 $\downarrow$   
 $AB - AC = 0$   
 $\downarrow$   
 $A(B - C) = 0$

$$A \left[ \underbrace{\underline{b}_1, \underline{b}_2, \underline{b}_3, \dots, \underline{b}_p}_{B} \right] = \left[ A\underline{b}_1, A\underline{b}_2, \dots, A\underline{b}_p \right]$$

$$\left( \begin{array}{c|c|c} -1 & 3 & 1 \\ 2 & 0 & 1 \\ 3 & -1 & 1 \end{array} \right) \left( \begin{array}{cccccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ -2 & 0 & 0 & 2 & 0 & 1 \end{array} \right) = \left( \begin{array}{cccccc} 0 & 3 & -1 & 2 & 2 & 1 \\ 0 & 0 & 2 & 2 & 2 & 1 \\ 0 & -1 & 3 & 2 & 2 & 1 \end{array} \right)$$

$L$

$$\left( \begin{array}{ccc} -1 & 3 & 1 \\ 2 & 0 & 1 \\ 3 & -1 & 1 \end{array} \right) \left( \begin{array}{ccc} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{array} \right) = \left( \begin{array}{ccc} 3 & 1 & -1 \\ 0 & 1 & 2 \\ -1 & 1 & 3 \end{array} \right)$$

↑ 3<sup>rd</sup> col  
2<sup>nd</sup> row

2<sup>nd</sup> row 3<sup>rd</sup> col

$$\left( \begin{array}{ccc} -1 & 3 & 1 \\ 2 & 0 & 1 \\ 3 & -1 & 1 \end{array} \right) \left( \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right) = \left( \begin{array}{ccc} -1 & 3 & 1 \\ 2 & 0 & 1 \\ 3 & -1 & 1 \end{array} \right)$$

$\underbrace{\text{identity } I_{3 \times 3}}$

$$\left( \begin{array}{cc} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{array} \right) + \left( \begin{array}{cc} -1 & 2 \\ 0 & -2 \\ 2 & 0 \end{array} \right) = \left( \begin{array}{cc} 0 & 3 \\ 2 & -1 \\ 5 & 1 \end{array} \right)$$

$$\alpha \left( \begin{array}{cc} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{array} \right) = \left( \begin{array}{cc} \alpha & \alpha \\ 2\alpha & \alpha \\ 3\alpha & \alpha \end{array} \right)$$

$\left( \begin{array}{cc} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{array} \right) \left( \begin{array}{cc} \alpha & 0 \\ 0 & \alpha \end{array} \right) = \left( \begin{array}{cc} \alpha & \alpha \\ 2\alpha & \alpha \\ 3\alpha & \alpha \end{array} \right)$

$\alpha I_{2 \times 2}$

The diagram shows two matrices, A and B, with their dimensions labeled. Matrix A has dimensions  $m \times n$  and m rows, n cols. Matrix B has dimensions  $n \times p$  and n rows, p cols. An arrow points from the intersection of the first row of A and the first column of B to the result matrix C, which has dimensions  $m \times p$ . The label 'm x p' is written below the arrow.

in general  $AB \neq BA$

$$\text{Distributive law : } A(B + C) = AB + AC$$

$$(A+B)C = AC + BC$$

$$\text{Associative law : } (A \circ B) \circ C = A \circ (B \circ C)$$

$$\neq A(c_B)$$

$$\underline{A}_1 \left( \begin{pmatrix} A_2 & \begin{pmatrix} A_3 & A_4 \end{pmatrix} \end{pmatrix} A_5 \right)$$

In particular  $A\underline{x} = \text{linear comb of cols of } A$ .

$$A = \begin{pmatrix} -1 & 3 & 1 \\ 2 & 0 & 1 \\ 3 & -1 & 1 \end{pmatrix} \quad \begin{matrix} b_1 = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} \\ b_2 = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} \end{matrix}$$

Is  $b$ , a linear combination of  $\begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$ ,  $\begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix}$  and  $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ .

$I_s \ b_2 \ a \quad - \parallel \quad \text{---} \quad n \quad \text{---}$

$$\underline{\text{Solve}} \quad A \underline{x} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

If there is a soln,  $\underline{b}_2$  is a linear comb of cols of A.

If there is NO soln,  $b_2$  is NOT a —

